Return operator, cross Bell bases and protocol of teleportation of arbitrary multipartite qubit entanglement

Zai-Zhe Zhong

Department of Physics, Liaoning Normal University, Dalian 116029, Liaoning, China. E-mail: zhongzaizheh@hotmail.com

In this paper, we define the return operator, the cross product operator and the cross Bell bases. Using the cross Bell bases, we give a protocol of teleportation of arbitrary multipartite qubit entanglement, this scheme is a quite natural generalization of the BBCJPW scheme. We find that this teleportation, in fact, is essentially determined by the teleportation of every single unknown qubit state as in the original scheme of BBCJPW. The calculation in detail is given for the case of tripartite qubit.

The quantum teleportation is a quite interesting and important topic in modern quantum mechanics and quantum information (about this topic and references, can see [1]). The original scheme of BBCJPW[2] is to discuss the teleportation of single unknown qubit pure-state, in which there is no teleportation problem of quantum entanglement of course. In view of application of quantum information theory, it obviously is not enough,, we must consider the problems of teleportation of multipartite quantum entangled states, first the multipartite qubit entangled states. Recently, there are some works (e.g. see [3-6]) discussing the problems of teleportation of arbitrary multipartite qubit entanglement, especially the bipartite and tripartite cases. However, in these schemes, generally, the mathematical form and concrete operation in physics seem more complex, the relation between them and BBCJPW scheme is not quite clear, we must find more perfect scheme. We find that the key of problems is the order of factors in the tensor products of the entangled state taken as channel. In this paper we give a new way to discuss the problem of teleportation of arbitrary multipartite qubit entanglement. First, in order to overcome the puzzle brought by order problem, we define a 'return operator' and the 'cross product' operator. Next we define what is the cross Bell basis. Using the cross Bell bases, we give a improved protocol of teleportation of arbitrary multipartite qubit entanglement, it is a very natural generalization of the original scheme of BBCJPW, the latter only is an extremity case (N=1) of the former. We find that this teleportation, in fact, is essentially determined by the teleportation of every single unknown qubit state as in the original scheme of BBCJPW[2]. In mathematical form our scheme also very simple. For the case of tripartite qubit, we make the concrete calculation.

The train of thought in this paper is as follows. In the following we write the Bell states of particles 1 and 2 as

$$| \phi_{12}^{\pm} \rangle = \frac{1}{\sqrt{2}} (| 0_1 0_2 \rangle \pm | 1_1 1_2 \rangle), \, | \psi_{12}^{\pm} \rangle = \frac{1}{\sqrt{2}} (| 0_1 1_2 \rangle \pm | 1_1 0_2 \rangle) \tag{1}$$

Now we look back the character of standard teleportation BBCJPW scheme[2]. Suppose that Alice holds a unknown state $| \varphi_3 \rangle = \alpha | 0_3 \rangle + \beta | 1_3 \rangle \left(|\alpha|^2 + |\beta|^2 = 1 \right)$, two particles 1, 2 are in Alice and Bob respectively, particles 1, 2 are in a Bell state (quantum channel), say $| \psi_{12}^{-} \rangle$. It is known that the realization of the standard teleportation, in fact, completely dependents on the decomposition of the total state $| \Psi_{total} \rangle = | \psi_{12}^{-} \rangle | \varphi_3 \rangle$ as

$$
\begin{aligned}
| \Psi_{total} \rangle &= | \psi_{12}^{-} \rangle | \varphi_3 \rangle = \frac{1}{\sqrt{2}} (| 0_1 1_2 \rangle - | 1_1 0_2 \rangle) (\alpha | 0_3 \rangle + \beta | 1_3 \rangle) \\
&= \frac{1}{\sqrt{2}} \{| 0_1 \rangle (\alpha | 1_2 0_3 \rangle + \beta | 1_2 1_3 \rangle) - | 1_1 \rangle (\alpha | 0_2 0_3 \rangle + \beta | 0_2 1_3 \rangle)\} \\
&= \frac{1}{\sqrt{2}} \left\{ \begin{array}{l} | 0_1 \rangle \left[-\alpha \frac{1}{\sqrt{2}} (| \psi_{23}^{+} \rangle - | \psi_{23}^{-} \rangle) - \beta \frac{1}{\sqrt{2}} (| \phi_{23}^{+} \rangle - | \phi_{23}^{-} \rangle) \right] \\ - | 1_1 \rangle \left[\alpha \frac{1}{\sqrt{2}} (| \phi_{23}^{+} \rangle + | \phi_{23}^{-} \rangle) - \beta \frac{1}{\sqrt{2}} (| \psi_{23}^{+} \rangle + | \psi_{23}^{-} \rangle) \right] \end{array} \right\} \\
&= \frac{1}{2} \left\{ \begin{array}{l} (-\beta | 0_1 \rangle - \alpha | 1_1 \rangle) | \phi_{23}^{+} \rangle + (\beta | 0_1 \rangle - \alpha | 1_1 \rangle) | \phi_{23}^{-} \rangle \\ + (-\alpha | 0_1 \rangle + \beta | 1_1 \rangle) | \psi_{23}^{+} \rangle + (\alpha | 0_1 \rangle + \beta | 1_1 \rangle) | \psi_{23}^{-} \rangle \end{array} \right\}
\end{aligned}
\tag{2}
$$

The rest steps are standard, i.e. to take a Bell measurement for particles 2, 3 by Alice, and the wave function will collapse with probability $\frac{1}{4}$ to one of $(-\beta | 0_1 \rangle - \alpha | 1_1 \rangle) | \phi_{23}^{+} \rangle$, $\cdots$, $(\alpha | 0_1 \rangle + \beta | 1_1 \rangle) | \psi_{23}^{-} \rangle$, etc. We see that the essence of this scheme are to use the decomposition of a product state $| ij \rangle$ by the Bell bases and the Bell measurement for particles 2, 3. We can figure it as follows

$$
\underset{|\varphi_3\rangle}{\underset{3,Alice}{\bigcirc}} \otimes \underset{2,Alice}{\bigcirc} \overset{|\psi_{23}^-\rangle}{=\!=\!=\!=} \underset{1,\,Bob}{\bigcirc} \quad \overset{\text{Bell measurement}}{\underset{\longrightarrow\longrightarrow\longrightarrow}{\overset{\text{for 2,3}}{\longrightarrow}}} \quad \overset{\text{collapse into one}}{\underset{3,Alice}{\bigcirc}} \overset{\text{Bell state}}{=\!=\!=\!=} \underset{2,Alice}{\bigcirc} \otimes \underset{1,Bob}{\overset{\text{result}}{\bigcirc}} \tag{3}
$$

where ==== denotes a Bell state. In this paper, the figure of our scheme of teleportation of a N-partite ($N \geqslant 2$) qubit entangled state $\mid \varphi_3^{(n)} \rangle = \sum_{i_1,\cdots,i_n=0,1} \alpha_{i_1\cdots i_n} \mid i_1\cdots i_n \rangle \left(\sum_{i_1,\cdots,i_n=0,1} |\alpha_{i_1\cdots i_n}|^2 = 1 \right)$ is as follows, which obviously is the generalization of the above figure

$$
\begin{pmatrix}
\text{Entangled state } \mid \varphi^{(N)} \rangle, \text{ Alice} \\
\bigcirc_{2N+1} \\
\bigcirc_{2N+N} \\
\vdots \\
\bigcirc_{3N}
\end{pmatrix}
\otimes
\begin{pmatrix}
\text{Alice} \qquad\qquad \text{Bob} \\
\bigcirc_{N+1} ==== \bigcirc_1 \\
\bigcirc_{N+2} ==== \bigcirc_2 \\
\vdots \qquad \vdots \qquad \vdots \\
\bigcirc_{2N} \;\; ==== \bigcirc_N
\end{pmatrix}
$$

$$
\begin{array}{c}
\text{Bell measurements for pairs} \\
\underrightarrow{(N+1,2N+1),\cdots,(2N,3N)}
\end{array}
\text{collapses into one of}
\begin{pmatrix}
\text{Alice} \qquad\qquad \text{Alice} \\
\bigcirc_{N+1} ==== \bigcirc_{2N+1} \\
\bigcirc_{N+2} ==== \bigcirc_{2N+2} \\
\vdots \qquad \vdots \qquad \vdots \\
\bigcirc_{2N} \;\; ==== \bigcirc_{3N}
\end{pmatrix}
\otimes
\begin{pmatrix}
\text{result, Bob} \\
\bigcirc_1 \\
\bigcirc_2 \\
\vdots \\
\bigcirc_N
\end{pmatrix}
\tag{4}
$$

where $\begin{pmatrix} ==== \\ ==== \\ \vdots \\ ==== \end{pmatrix}$ denotes some 'cross Bell bases' (see below).

The concrete steps are as follows. In the first place, we need to define the return operator and the cross product operator ∇.

Definition 1. The so-called 'return operator' $\mathcal{R}$ is a linear operator, its action is to return the factor order of every term (a tensor product of single qubit state) of a N-partite qubit pure-state $\mid \Psi \rangle$ into the natural order as $\mid i_1 i_2 \cdots i_N \rangle$.

Example. $\mathcal{R} \left(a \mid 0_2 0_1 0_3 \cdots 0_N \rangle + b \mid 1_3 0_2 1_1 0 \cdots 0_N \rangle \right) = a \mid 0_1 0_2 0_3 \cdots 0_N \rangle + b \mid 1_1 0_2 1_3 0 \cdots 0_N \rangle$.

Obviously the operator $\mathcal{R}$ does not break the normalization and orthogonality, i.e. if $\mid \Psi \rangle$ is normal, then $\mathcal{R} (\mid \Psi \rangle)$ is also, and If $\langle \Phi \mid \Psi \rangle = 0$, then $\langle \mathcal{R} (\Phi) \mid \mathcal{R} (\mid \Psi \rangle) \rangle = 0$.

Definition 2. If $\left\{ \mid \Psi_{(n)} \rangle \right\} (n = 1, \cdots, N)$ is a sequence of N qubit pure-states $\mid \Psi_{(n)} \rangle = \sum_{i_n,j_n=0,1} c_{i_n j_n} \mid i_n j_n \rangle$, the cross product of (ordered) finite sequence $\left\{ \mid \Psi_{(n)} \rangle \right\}$ written by notation $\nabla \left(\mid \Psi_{(1)} \rangle, \cdots, \mid \Psi_{(n)} \rangle \right)$, which is a 2N-partite qubit pure-state, is defined to be the result $\mathcal{R} \left(\otimes_{n=1}^{N} \mid \Psi_{(n)} \rangle \right)$ of the ordinary tensor product $\otimes_{n=1}^{N} \mid \Psi_{(n)} \rangle = \sum_{i_1,\cdots,i_N} \left(\prod_{n=1}^{N} c_{i_n j_n} \right) \mid i_1 j_1 \cdots i_N j_N \rangle$, where $\mathcal{R}$ is the return operator with respect to the order $(i_1, \cdots, i_N, j_1, \cdots, j_N)$, i.e.

$$
\nabla \left(\mid \Psi_{(1)} \rangle, \cdots, \mid \Psi_{(n)} \rangle \right) = \sum_{i_1,\cdots,i_N} \left(\prod_{n=1}^{N} c_{i_n j_n} \right) \mid i_1 \cdots i_N j_1 \cdots j_N \rangle
\tag{5}
$$

We notice that although $\nabla \left(\mid \Psi_{(1)} \rangle, \cdots, \mid \Psi_{(n)} \rangle \right)$, in fact, has the completely same physical contents to $\otimes_{n=1}^{N} \mid \Psi_{(n)} \rangle$, in our scheme the expression $\nabla \left(\mid \Psi_{(1)} \rangle, \cdots, \mid \Psi_{(n)} \rangle \right)$ for the order $(i_1, \cdots, i_N, j_1, \cdots, j_N)$ is very important, it is just the reason to introduce the cross product operator ∇ in this paper. Obviously, the operator ∇ is linear for every $\mid \Psi_{(i)} \rangle$, e.g. $\nabla \left(\alpha \mid \Psi_{(1)} \rangle + \beta \mid \Psi'_{(1)} \rangle, \mid \Psi_{(2)} \rangle \cdots, \mid \Psi_{(n)} \rangle \right) = \alpha \nabla \left(\mid \Psi_{(1)} \rangle, \mid \Psi_{(2)} \rangle, \cdots, \mid \Psi_{(n)} \rangle \right) + \beta \nabla \left(\mid \Psi'_{(1)} \rangle, \mid \Psi_{(2)} \rangle, \cdots, \mid \Psi_{(n)} \rangle \right), \cdots$, etc.

Now by use of cross products, we can construct a basis of the Hilbert space $H_1 \otimes H_2 \otimes \cdots \otimes H_{2N}$, where every $H_r (r = 1, \cdots, 2N)$ is a Hilbert space of qubit states. For the sake of convenience, in the following we simply write the Bell bases as

$$
\mid \Omega_{1,2}^{(1)} \rangle \equiv \mid \phi_{12}^+ \rangle, \mid \Omega_{1,2}^{(2)} \rangle \equiv \mid \phi_{12}^- \rangle, \mid \Omega_{1,2}^{(3)} \rangle \equiv \mid \psi_{12}^+ \rangle, \mid \Omega_{1,2}^{(4)} \rangle \equiv \mid \psi_{12}^- \rangle
\tag{6}
$$

Definition 3. Let the set $\mathbb{B}_{1,\cdots,2N}$ be defined by

$$
\mathbb{B}_{1,\cdots,2N} = \left\{ \mathbb{B}_{1,\cdots,2N}^{(\lambda_1,\lambda_2,\cdots\lambda_N)} \text{ for all possible } \lambda_1, \lambda_2, \cdots, \lambda_N = 1, 2, 3, 4 \right\}
$$

$$
\mathbb{B}_{1,\cdots,2N}^{(\lambda_1,\lambda_2,\cdots\lambda_N)} \equiv \nabla \left(\mid \Omega_{1,n+1}^{(\lambda_1)} \rangle, \mid \Omega_{2,n+2}^{(\lambda_2)} \rangle, \cdots, \mid \Omega_{N,2N}^{(\lambda_N)} \rangle \right)
\tag{7}
$$

It is easily verified that $\mathbb{B}_{1,\cdots,2N}$ has 2^{4N} entries and surly forms a complete orthogonal basis of $H_1 \otimes H_2 \otimes \cdots \otimes H_{2N}$, we call it the 'cross Bell basis', which is a generalization of the ordinary Bell bases. Here, it must be stressed that it is different from the ordinary Bell bases that **all cross Bell bases for any $N \geqslant 3$ are not maximal entangled states**. In fact, they obviously are entangled, but partially separable[7,8] states, e.g., all $\mathbb{B}_{1234}^{(r,s)}$ are 13-24 entangled, etc. In the following, we shall take one entry of $\mathbb{B}_{1,\cdots,2N}$ as the quantum channel, this fact also identifies just with the view of [9].

These bases will construct the parts described by $====$ as the figure as in Eq.(4). By use of the following equations

$$| 0_1 0_2 \rangle = \frac{1}{\sqrt{2}} \left(| \phi_{12}^+ \rangle + | \phi_{12}^- \rangle \right), | 1_1 1_2 \rangle = \frac{1}{\sqrt{2}} \left(| \phi_{12}^+ \rangle - | \phi_{12}^- \rangle \right)$$

$$| 0_1 1_2 \rangle = \frac{1}{\sqrt{2}} \left(| \psi_{12}^+ \rangle + | \psi_{12}^- \rangle \right), | 1_1 0_2 \rangle = \frac{1}{\sqrt{2}} \left(| \psi_{12}^+ \rangle - | \psi_{12}^- \rangle \right) \tag{8}$$

we obtain the transformation relation between the natural basis and cross Bell basis as follows

$$| 0_1 0_2 \cdots 0_{2N-1} 0_{2N} \rangle = 2^{-\frac{N}{2}} \nabla \left(| \Omega_{1,N+1}^{(1)} \rangle + | \Omega_{1,N+1}^{(2)} \rangle, \cdots, | \Omega_{N,2N}^{(1)} \rangle + | \Omega_{N,2N}^{(2)} \rangle \right)$$

$$= 2^{-\frac{N}{2}} \sum_{\lambda_1,\lambda_2,\cdots,\lambda_N=1,2,} \mathbb{B}_{1,\cdots,2N}^{(\lambda_1,\lambda_2,\cdots\lambda_N)}$$

$$| 0_1 0_2 \cdots 0_{2N-1} 1_{2N} \rangle = 2^{-\frac{N}{2}} \nabla \left(| \Omega_{1,N+1}^{(1)} \rangle + | \Omega_{1,N+1}^{(2)} \rangle, \cdots, | \Omega_{N-1,2N-1}^{(1)} \rangle + | \Omega_{N-1,2N-1}^{(2)} \rangle, | \Omega_{N,2N}^{(3)} \rangle + | \Omega_{N,2N}^{(4)} \rangle \right)$$

$$= 2^{-\frac{N}{2}} \sum_{\lambda_1,\lambda_2,\cdots,=1,2.\ \lambda_N=3,4} \mathbb{B}_{1,\cdots,2N}^{(\lambda_1,\lambda_2,\cdots\lambda_N)}$$

$$| 0_1 0_2 \cdots 1_{2N-1} 0_{2N} \rangle = 2^{-\frac{N}{2}} \nabla \left(\begin{array}{c} | \Omega_{1,N+1}^{(1)} \rangle + | \Omega_{1,N+1}^{(2)} \rangle, \cdots, | \Omega_{N-2,2N-2}^{(1)} \rangle + | \Omega_{N-2,2N-2}^{(2)} \rangle, \\ | \Omega_{N-1,2N-1}^{(3)} \rangle - | \Omega_{N-1,2N-1}^{(4)} \rangle, | \Omega_{N,2N}^{(1)} \rangle + | \Omega_{N,2N}^{(2)} \rangle \end{array} \right)$$

$$= (-1)^{\lambda_{N-1}+1} 2^{-\frac{N}{2}} \sum_{\lambda_1,\lambda_2,\cdots,\lambda_{N-2},\lambda_N=1,2.\ \lambda_{N-1}=3,4} \mathbb{B}_{1,\cdots,2N}^{(\lambda_1,\lambda_2,\cdots\lambda_N)}$$

$$\cdots\cdots\cdots\cdots \tag{9}$$

$$| 1_1 1_2 \cdots 1_{2N-1} 0_{2N} \rangle = 2^{-\frac{N}{2}} \nabla \left(| \Omega_{1,N+1}^{(1)} \rangle - | \Omega_{1,N+1}^{(2)} \rangle, \cdots, | \Omega_{N-1,2N-11}^{(1)} \rangle - | \Omega_{N-1,2N+1}^{(2)} \rangle, | \Omega_{N,2N}^{(3)} \rangle - | \Omega_{N,2N}^{(4)} \rangle \right)$$

$$= (-1)^{\sum_{s=1}^{N} \lambda_s - N} 2^{-\frac{N}{2}} \sum_{\lambda_1,\lambda_2,\cdots,=1,2.\ R_N=3,4} \mathbb{B}_{1,\cdots,2N}^{(\lambda_1,\lambda_2,\cdots\lambda_N)}$$

$$| 1_1 1_2 \cdots 1_{2N-1} 1_{2N} \rangle = 2^{-\frac{N}{2}} \nabla \left(| \Omega_{1,N+1}^{(1)} \rangle - | \Omega_{1,N+1}^{(2)} \rangle, \cdots, | \Omega_{N,2N}^{(1)} \rangle - | \Omega_{N,2N}^{(2)} \rangle \right)$$

$$= (-1)^{\sum_{s=1}^{N} \lambda_s - N} 2^{-\frac{N}{2}} \sum_{\lambda_1,\lambda_2,\cdots,\lambda_N=1,2} \mathbb{B}_{1,\cdots,2N}^{(\lambda_1,\lambda_2,\cdots\lambda_N)}$$

We still need to use the following common relations

$$| \phi_{12}^\pm \rangle | 0_3 \rangle = \frac{1}{2} \left[| 0_1 \rangle \left(| \phi_{23}^+ \rangle + | \phi_{23}^- \rangle \right) \pm | 1_1 \rangle \left(| \psi_{23}^+ \rangle - | \psi_{23}^- \rangle \right) \right]$$

$$| \phi_{12}^\pm \rangle | 1_3 \rangle = \frac{1}{2} \left[| 0_1 \rangle \left(| \psi_{23}^+ \rangle + | \psi_{23}^- \rangle \right) \pm | 1_1 \rangle \left(| \phi_{23}^+ \rangle - | \phi_{23}^- \rangle \right) \right]$$

$$| \phi_{12}^\pm \rangle | 0_3 \rangle = \frac{1}{2} \left[| 0_1 \rangle \left(| \psi_{23}^+ \rangle - | \psi_{23}^- \rangle \right) \pm | 1_1 \rangle \left(| \phi_{23}^+ \rangle + | \phi_{23}^- \rangle \right) \right] \tag{10}$$

$$| \phi_{12}^\pm \rangle | 1_3 \rangle = \frac{1}{2} \left[| 0_1 \rangle \left(| \phi_{23}^+ \rangle - | \phi_{23}^- \rangle \right) \pm | 1_1 \rangle \left(| \psi_{23}^+ \rangle + | \psi_{23}^- \rangle \right) \right]$$

They can be simply reduced as $(i = 0, 1)$

$$| \Omega_{12}^{(\lambda)} \rangle | i_3 \rangle = \frac{1}{2} \left[\begin{array}{c} | 0_1 \rangle \left(| \Omega_{23}^{(1+2i)} \rangle + | \Omega_{23}^{(2+2i)} \rangle \right) \\ + (-1)^{\lambda+1} | 1_1 \rangle \left(| \Omega_{23}^{(3-2i)} \rangle - | \Omega_{23}^{(4-2i)} \rangle \right) \end{array} \right] \text{ for } \lambda = 1, 2$$

$$| \Omega_{12}^{(\lambda)} \rangle | i_3 \rangle = \frac{1}{2} \left[\begin{array}{c} | 0_1 \rangle \left(| \Omega_{23}^{(3+2i \bmod 4)} \rangle - | \Omega_{23}^{(4+2i \bmod 4)} \rangle \right) + \\ (-1)^{\lambda+1} | 1_1 \rangle \left(| \Omega_{23}^{(1+2i \bmod 4)} \rangle + | \Omega_{23}^{(2+2i \bmod 4)} \rangle \right) \end{array} \right] \text{ for } \lambda = 3, 4 \tag{11}$$

3

Now, by use of the above cross Bell bases and relations, we can realize the teleportation of an unknown N-partite qubit entangled state. For the sake of simplicity, we only concretely discuss the case of tripartite qubit states. As for the simplest case of N=2, the final result is same to [3], however in the latter the way is more complex; For higher dimensional cases, it is similar, only the calculation becomes more complex. We take a cross Bell basic entry, say $\mathbb{B}_{1,2,3,4,5,6}^{(4,4,4)} = \nabla\left(\psi_{14}^-, \psi_{25}^-, \psi_{36}^-\right)$, as the quantum channel, i.e. particles 1,2,3 are in Bob, particles 4,5,6 are in Alice and 1,2,3,4,5,6 are in the entangled state $\mathbb{B}_{1,2,3,4,5,6}^{(4,4,4)} = \nabla\left(\psi_{14}^-, \psi_{25}^-, \psi_{36}^-\right)$. Suppose that Alice holds particles 7,8,9 which are in an unknown entangled state as $\mid \varphi_{789}\rangle = \sum_{i_7,i_8,i_9=0,1} \alpha_{i_7 i_8 i_9} \mid i_7 i_8 i_9\rangle$, where $\sum_{i_7,i_8,i_9=0,1} \left|\alpha_{i_7 i_8 i_9}\right|^2 = 1$. The total wave function is

$$
\begin{aligned}
\mid \Psi_{123456789}\rangle &= \mathbb{B}_{1,2,3,4,5,6}^{(4,4,4)} \otimes \mid \varphi_{789}\rangle = \mathcal{R}\left(\psi_{14}^- \otimes \psi_{25}^- \otimes \psi_{36}^- \otimes \mid \varphi_{789}\rangle\right) \\
&= \sum_{i_7,i_8,i_9=0,1} \alpha_{i_7 i_8 i_9} \mathcal{R}\left(\left[\psi_{14}^- \otimes \mid i_7\rangle\right] \otimes \left[\psi_{25}^- \otimes \mid i_8\rangle\right] \otimes \left[\psi_{36}^- \otimes \mid i_9\rangle\right]\right)
\end{aligned}
\tag{12}
$$

where $\mathcal{R}$ denotes the above return operation with respect to the natural order $(1, 2, \cdots, 9)$. Using Eqs.(8-11), we obtain

$$
\begin{aligned}
\mid \Psi_{123456789}\rangle = \frac{1}{8} &\left\{\alpha_{0_7 0_8 0_9} \mathcal{R}\left[\begin{array}{c} \mid 0_1\rangle\left(\psi_{47}^+ - \psi_{47}^-\right) - \mid 1_1\rangle\left(\phi_{47}^+ + \phi_{47}^-\right) \\ \otimes \mid 0_1\rangle\left(\psi_{58}^+ - \psi_{58}^-\right) - \mid 1_1\rangle\left(\phi_{58}^+ + \phi_{58}^-\right) \\ \otimes \mid 0_1\rangle\left(\psi_{69}^+ - \psi_{69}^-\right) - \mid 1_1\rangle\left(\phi_{69}^+ + \phi_{69}^-\right) \end{array}\right]\right\} \\
+ \alpha_{0_7 0_8 1_9} &\mathcal{R}\left[\begin{array}{c} \mid 0_1\rangle\left(\psi_{47}^+ - \psi_{47}^-\right) - \mid 1_1\rangle\left(\phi_{47}^+ + \phi_{47}^-\right) \\ \otimes \mid 0_1\rangle\left(\psi_{58}^+ - \psi_{58}^-\right) - \mid 1_1\rangle\left(\phi_{58}^+ + \phi_{58}^-\right) \\ \otimes \mid 0_1\rangle\left(\phi_{69}^+ - \phi_{69}^-\right) - \mid 1_1\rangle\left(\psi_{69}^+ + \psi_{69}^-\right) \end{array}\right] + \cdots \\
+ \alpha_{1_7 1_8 1_9} &\mathcal{R}\left[\begin{array}{c} \mid 0_1\rangle\left(\phi_{47}^+ - \phi_{47}^-\right) - \mid 1_1\rangle\left(\psi_{47}^+ + \psi_{47}^-\right), \\ \mid 0_1\rangle\left(\phi_{58}^+ - \phi_{58}^-\right) - \mid 1_1\rangle\left(\psi_{58}^+ + \psi_{58}^-\right), \\ \mid 0_1\rangle\left(\phi_{69}^+ - \phi_{69}^-\right) - \mid 1_1\rangle\left(\psi_{69}^+ + \psi_{69}^-\right) \end{array}\right]
\end{aligned}
\tag{13}
$$

Make carefully collocation by the definition 2, we find

$$
\mid \Psi_{123456789}\rangle = \frac{1}{8} \sum_{R,S,T=1}^4 \mid \varphi'_{(rst)}\rangle \otimes \mathbb{B}_{4,5,6,7,8,9}^{(R,S,T)}, \quad \mid \varphi'_{(R,S,T)}\rangle = U_{RST} \mid \varphi_{123}\rangle
\tag{14}
$$

where $\mid \varphi_{123}\rangle \equiv$ the result of substitution $\mid i_{r+6}\rangle \longrightarrow \mid i_r\rangle$ $(r = 1,2,3$ and $i = 0,1)$ from $\mid \varphi_{789}\rangle$ (notice that α_{RST} does not change), i.e. $\mid \varphi_{123}\rangle = \sum_{i,j,k=0,1} \alpha_{i_7 j_8 k_9} \mid i_1 j_2 k_3\rangle$, and U_{RST} $(R, S, T = 1, 2, 3, 4)$ are sixty-four 8×8 unitary matrices

$$
\begin{aligned}
U_{RST} &= U_R \otimes U_S \otimes U_T, \\
U_1 = U_{\phi^+} &= i\sigma_y = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, U_2 = U_{\phi^-} = -\sigma_x = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \\
U_3 = U_{\psi^+} &= \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, U_4 = U_{\psi^-} = -\sigma_0 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}
\end{aligned}
\tag{15}
$$

σ_0 is the unit matrix, and $\sigma_x, \sigma_y, \sigma_z$ are the ordinary Pauli matrices. It is very interesting that U_R $(R = 1, 2, 3, 4)$ are just those unitary matrices appearing in the BBCJPW scheme[2] when $\mid \psi_{12}^-\rangle$ is taken as channel. The rest steps are standard, i. e. Alice, respectively, makes three Bell measurements for particle pairs $(4,7)$, $(5,8)$ and $(6,9)$, she will obtain one of $\left\{\mathbb{B}_{456789}^{(R,S,T)}\right\}$ with equal probability $\frac{1}{64}$, simultaneously Bob obtain a corresponding $\mid \varphi'_{(RST)}\rangle \mid$. When Alice informs her measurement to Bob by classical communication, then Bob at once knows the correct result must be $\mid \varphi_{123}\rangle = U_{RST}^{-1} \mid \varphi'_{(RST)}\rangle = U_R^{-1} \otimes U_S^{-1} \otimes U_T^{-1} \mid \varphi'_{(RST)}\rangle$, etc., so the teleportation of N-partite qubit entangled state $\mid \varphi_{789}\rangle$ from Alice to Bob is completed.

If we take other $\mathbb{B}_{456789}^{(R,S,T)}$ as the quantum channels, the results are similar. It can be verified that the above method can be directly generalized to arbitrary unknown N-partite qubit entangled states, i.e. if we take a $\mathbb{B}_{1\cdots2N}^{(R_1,\cdots,R_N)}$ $(R_1, \cdots, R_N = 1, \cdots, N)$ as quantum channel, and Alice holds an arbitrary unknown N-partite qubit entangled state $\mid \varphi_{(2N+1)\cdots3N}\rangle = \sum_{i_{2N+1},\cdots,i_{3N}=0,1} \alpha_{i_{2N+1}\cdots i_{3N}} \mid i_{2N+1}\cdots i_{3N}\rangle$, she makes N Bell measurements for particle pairs $(N+1, 2N+1), \cdots, (2N, 3N)$, then Alice must obtain one of $\left\{\mathbb{B}_{N+1,\cdots,3N}^{(S_1,\cdots,S_N)}\right\}$ with equal probability 2^{-2N}, and Bob obtain a state $\mid \varphi'_{1\cdots N}\rangle = U_{S_1,\cdots,S_N}^{(R_1,\cdots,R_N)} \mid \varphi'_{(R_1,\cdots,R_N)}\rangle$, where $\mid \varphi'_{(R_1,\cdots,R_N)}\rangle = \sum_{i_{2N+1},\cdots,i_{3N}=0,1}$

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$\alpha_{i_{2N+1}\cdots i_{3N}} \mid i_1 \cdots i_N\rangle$. Especially we find that if in BBCJPW scheme the unitary matrix corresponding to channel $\mid \Omega_{k,N+k}^{(R_k)}\rangle (R = 1,2,3,4)$ is $U_{S_{R_k}}^{(R_k)}$ (i.e. a unknown state $\mid \varphi_{2N+k}\rangle = \alpha \mid 0_{2N+k}\rangle + \beta \mid 1_{2N+k}\rangle$ is in Alice, when Alice makes joint measurement for particle pair $(N+k, 2N+k)$, the total wave function $\mid \Omega_{k,N+k}^{(R_k)}\rangle \mid \varphi_{2N+k}\rangle$ collapses to one $\left(U_{S_{R_k}}^{(R_k)} \mid \varphi_k'\rangle\right) \otimes \mid \Omega_{2,3}^{(S_{R_k})}\rangle (\mid \varphi_k'\rangle = \alpha \mid 0_{k1}\rangle + \beta \mid 1_k\rangle,\ S_{R_k} = 1,2,3,4)$ with equal probability $\frac{1}{4}$), then in our scheme

$$U_{S_1,\cdots,S_N}^{(R_1,\cdots,R_N)} = \otimes_{k=1}^{N} U_{S_{R_k}}^{(R_k)} \tag{16}$$

holds and Bob knows the correct result must be $\mid \varphi_{1\ldots N}\rangle = \left(\otimes_{k=1}^{N} \left(U_{S_{R_k}}^{(R_k)}\right)^{-1}\right) \mid \varphi_{R_1,\cdots,R_N}'\rangle$. This conclusion tall us that **the teleportation of arbitrary multipartite qubit entanglement, in fact, is essentially determined by the teleportation of every single unknown qubit state.** In fact, this is necessary. In fact, the entanglement of the state must invariant in any teleportation, this means that the above teleportation must become N independent teleportation for a separable state $\mid \varphi_{(2N+1)\cdots 3N}\rangle = \otimes_{s=1}^{N} \mid \varphi_{2N+s}\rangle$, where every $\mid \varphi_{2N+s}\rangle = \alpha_{2N+s} \mid 0_{2N+s}\rangle + \beta_{2N+s} \mid 1_{2N+s}\rangle (s = 1,\cdots,N)$ is a qubit pure-state, i.e. Eq.(16) must hold for $\mid \varphi_{(2N+1)\cdots 3N}\rangle$ (where $U_{S_{R_k}}^{(R_k)}$ correponds to $\mid \varphi_{2N+k}\rangle$). Therefore Eq. (16) generally hold, because every $U_{S_1,\cdots,S_N}^{(R_1,\cdots,R_N)}$ is only dependent on indices $(R_1,\cdots,R_N)$, $(S_1,\cdots,S_N)$.

Conclusion. We give a general scheme of teleportation of arbitrary multipartite qubit entanglement. The return operators and cross product operators are convenient in this scheme. Every one of cross Bell bases can be taken as the quantum channel to realize the teleportation of arbitrary multipartite qubit entanglement, this teleportation is essentially determined by the teleportation of every single unknown qubit state.

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